

## ACCURATE AND DETAILED FDTD SOLUTION OF MICROWAVE INTEGRATED CIRCUITS USING THE FREQUENCY SHIFTING TECHNIQUE

An Ping Zhao and Antti V. Räisänen

Radio Laboratory  
Helsinki University of Technology, FIN-02150, Espoo, Finland

### Abstract

By employing the elementary property (i.e., the frequency shifting technique) of the Fourier transform, a simple and efficient spectral estimation technique is developed. With this technique, extremely accurate and detailed frequency domain scattering parameters of general microwave integrated circuits can be obtained, without increasing any additional computation and programming efforts in the original FDTD simulation.

### Introduction

Since the finite difference time domain (FDTD) method was introduced by Yee in 1966 [1], it has been proven to be one of the most prominent computer tools for investigating the characteristics of microwave integrated circuits. In the conventional FDTD method, the frequency increment used in the Discrete Fourier Transform (DFT) is given through the condition  $\Delta f_0 = 1/(N\Delta t)$ , where  $\Delta t$  and  $N$  are the time increment and number of iterations used in the FDTD simulation, respectively. Because of the above condition, a quite large number of iterations is often required to investigate the resonant behavior of narrow-band microwave circuits. This certainly results in a major drawback to the conventional FDTD implementation, i.e., enormous computation time and computer memory are needed. To overcome this drawback, the

following spectral estimation approaches have been developed: the Prony's method [2], the auto-regressive method [3], and the adaptive sampling technique combined with the Prony's method [4]. Among the above approaches [2-4], the implementation of the approach proposed in [4] is easier. However, this approach [4] has the following two disadvantages: (i) the scattering parameters obtained with this approach at the additional frequency points are less accurate because a relatively small number of the temporal samples, compared with the number of iterations needed for reaching the 'numerical' steady-state of the system, is used in the DFT operation; (ii) it cannot be used to improve the accuracy of the scattering parameters at frequencies lower than  $3\Delta f_0$  due to the convergence condition required by the approach [4].

To overcome the above disadvantages of the approach [4], in this paper an accurate and detailed FDTD solution technique (based on the elementary property of the Fourier transform) for improving the efficiency of FDTD analysis of general microwave integrated circuits is developed. With this technique, very accurate and detailed scattering parameters in the whole frequency range of interest can be simply obtained by giving an initial frequency shift in the frequency domain. The accuracy and efficiency of the proposed technique are demonstrated by applying it to a strongly resonating narrow-band microwave circuit.

WE  
2B

## Theory

Suppose that the discrete time series supplied by the FDTD simulation is  $x(n\Delta t)$ , where  $n = 0, 1, 2, \dots, N-1$ . This discrete time series is transformed into the frequency domain as:

$$\begin{aligned} X_1(2\pi f_1) &= \frac{1}{N\Delta t} \sum_{n=0}^{N-1} x(n\Delta t) e^{-i(2\pi f_1)\Delta t} \Delta t \\ &= \frac{1}{N} \sum_{n=0}^{N-1} x(n\Delta t) e^{-i2\pi f_1 \Delta t} \end{aligned} \quad (1)$$

where  $i = \sqrt{-1}$  is the imaginary unit and  $f_1 = j\Delta f_0$  is the frequency points in the frequency domain. From the elementary property of the Fourier transform [5], Eq. (1) can be rewritten as the following after giving an arbitrary initial frequency shift,  $f_{\text{shift}}$ , in the frequency domain:

$$X_1(2\pi(f_1 + f_{\text{shift}})) = \frac{1}{N} \sum_{n=0}^{N-1} x(n\Delta t) e^{-i2\pi(f_1 + f_{\text{shift}})\Delta t} \quad (2)$$

Assume that, to achieve a desired spectral resolution in the whole frequency range of interest,  $M$  ( $\geq 1$ ) additional frequency points are needed between the original frequency points  $j\Delta f_0$  and  $(j+1)\Delta f_0$ . Therefore, the  $m$ -th frequency shift,  $f_{\text{shift}}^m$ , can be formed as:

$$f_{\text{shift}}^m = \frac{m((j+1)\Delta f_0 - j\Delta f_0)}{M+1} = \frac{m\Delta f_0}{M+1} \quad (3)$$

where  $m = 1, 2, \dots, M$ ; and the frequencies  $(f_1 + f_{\text{shift}}^m)$  are the additional frequency points created by the frequency shifting technique. Combining Eqs. (3) and (2), one obtains:

$$X_{1,m}\left(2\pi\left(f_1 + \frac{m\Delta f_0}{M+1}\right)\right) = \frac{1}{N} \sum_{n=0}^{N-1} x(n\Delta t) e^{-i2\pi\left(f_1 + \frac{m\Delta f_0}{M+1}\right)\Delta t} \quad (4)$$

Eq. (4) is the final formula for calculating the S-parameters. Comparing (4) with (1), one can see

that with the frequency shifting FDTD technique the S-parameters at the additional frequency points  $f_1 + m\Delta f_0/(M+1)$  can be obtained, although with the conventional FDTD/DFT method they can be obtained *only* at some certain frequency points  $f_j$ . Moreover, it can be seen from (4) and (1) that the scattering parameters obtained at the additional frequency points with the frequency shifting FDTD technique have the same accuracy as those obtained at the certain frequency points with the conventional FDTD/DFT method, because (for both cases) at all frequency points the same number of the temporal samples (i.e.,  $x(n\Delta t)$ ) is used in the DFT operation. Therefore, in contrast with [4], not only detailed but also accurate results can be obtained by the frequency shifting FDTD technique. In addition, it should be noted that (4) reduces to (1) when  $m = 0$  for any given  $M$ . This means that in this case the frequency shifting FDTD method becomes the conventional FDTD/DFT method.

In summary, to obtain the accurate and detailed numerical results with the frequency shifting FDTD method, the following steps are involved: (a) Run the conventional FDTD simulation until the ‘numerical’ steady-state of the system under consideration is reached; (b) Divide the original frequency interval  $(j\Delta f_0, (j+1)\Delta f_0)$  into  $M+1$  segments to create the additional frequency points in the frequency domain; (c) Perform the DFT operation  $(M+1)$  times, i.e., perform the DFT for each  $m$  (where  $m = 0, 1, 2, \dots, M$ ), and store the data; (d) Plot the completed scattering parameters against frequency using the data obtained from all the DFTs.

## Numerical Results

To validate the proposed frequency shifting FDTD technique, we apply it to a strongly resonating narrow-band structure - the microstrip low-pass filter [6,7]. The discrete time series (i.e.,  $x(n\Delta t)$ ) used here for calculating the scattering parameters is taken from case 1 in [6], and the

important parameters used in the FDTD simulation of the microstrip low-pass filter were [6,7]: the time increment  $\Delta t = 0.441$  ps, the number of iterations  $N = 4000$ , and thus the frequency increment  $\Delta f_0 = 0.566893$  GHz. Fig. 1 shows the numerical results for  $M$  with different values, and the numerical results with  $m = 0$  is obtained from the conventional FDTD/DFT method. As shown in Fig. 1, more accurate and detailed results within the whole frequency range of interest can be obtained when  $M$  is increased. In addition, it can be seen from Fig. 1 that the accuracy of the results (particularly for  $|S_{11}|$ ) obtained with the conventional FDTD/DFT method (i.e.,  $m = 0$ ) at frequencies below  $3\Delta f_0$  ( $\approx 1.7$  GHz) also need to be improved because of the sharp variation in the S-parameters. How the accuracy of  $|S_{11}|$  (also  $|S_{21}|$ ) within the frequency range of  $0 \sim 3\Delta f_0$  is improved with the frequency shifting FDTD technique is also shown in Fig. 1. As mentioned before, however, no improvement on the results within such a frequency range (i.e.,  $0 \sim 3\Delta f_0$ ) can be achieved by the approach proposed in [4]. To confirm the accuracy of the results obtained with the proposed technique, a comparison between the results obtained with the conventional FDTD/DFT method ( $N = 24000$  and  $m = 0$ ) and the proposed technique ( $N = 4000$  and  $M = 5$ ) is shown in Fig. 2. As shown in Fig. 2, the maximum difference between these two calculations appears in  $|S_{21}|$  at  $f = 7.55858$  GHz, and it is 0.36912 dB. However, at this frequency the relative difference is less than 1.0%. In addition, to further demonstrate the convergence behavior for the S-parameters in the frequency range of 6.708 GHz to 8.22 GHz, the results for  $M$  with further increments are shown in Fig. 3. The results plotted in Fig. 3 indicate that extremely accurate and detailed results can be obtained by simply increasing the value of  $M$ .

## Conclusions

By introducing the elementary property (i.e., the frequency shifting technique) of the Fourier trans-

form into the conventional FDTD/DFT method, a simple and efficient spectral estimation technique has been developed. With this technique, very accurate and detailed scattering parameters within the whole frequency range of interest can be easily obtained by giving initial frequency shifts in the frequency domain. The validity and efficiency of the proposed spectral estimation technique have been confirmed in this paper. Compared with other existing techniques for enhancing the efficiency of the conventional FDTD method, the capability and efficiency are strongly enhanced, in a very simple manner, by the proposed technique. Therefore, the proposed approach would provide a very efficient, general-purpose EM simulation tool for characterizing general microwave integrated circuits, including strongly resonating narrow-band systems.

## References

- [1] K. S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations on isotropic media," IEEE Trans. Antennas Propagat., Vol. AP-14, pp. 302-307, 1966.
- [2] W. L. Ko and R. Mittra, "A combination of FD-TD and Prony's methods for analyzing microwave integrated circuits," IEEE Trans. Microwave Theory Tech., Vol. MTT-39, pp. 2176-2181, 1991.
- [3] J. Litva, C. Wu, K. L. Wu, and J. Chen, "Some considerations for using the finite difference time domain technique to analyze microwave integrated circuits," IEEE Microwave and Guided Wave Lett., Vol. 3, pp. 438-440, 1993.
- [4] X. P. Lin and K. Naishadham, "A spectral estimation technique to improve the efficiency of FDTD method for narrow-band microwave circuits," 1995 IEEE MTT-S Int. Microwave Symp. Digest, pp. 1015-1018.
- [5] E. O. Brigham, "The Fast Fourier Transform And Its Applications," Prentice-Hall, 1988.

[6] A. P. Zhao, A. V. Räisänen, and S. R. Cvetkovic, "A fast and efficient FDTD algorithm for the analysis of planar microstrip discontinuities by using a simple source excitation scheme," IEEE Microwave and Guided Wave Lett., Vol. 5, pp. 341-343, 1995.

[7] D. M. Sheen, S. M. Ali, M. D. Abouzahra, and J. A. Kong, "Application of the three-dimensional finite-difference time-domain method to the analysis of planar microstrip circuits," IEEE Trans. Microwave Theory Tech., Vol. MTT-38, pp. 849-857, 1990.

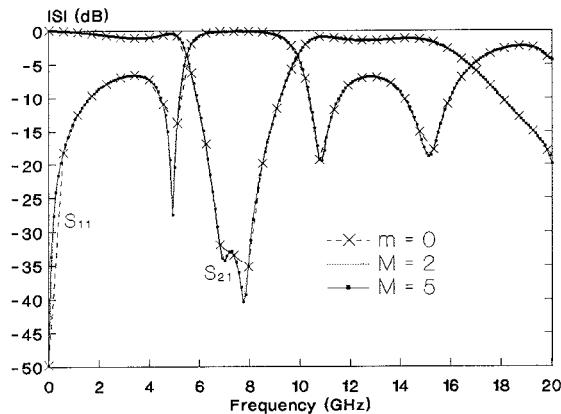


Fig. 1 S-parameters of the microstrip low-pass filter with different values of  $M$ .  $N = 4000$  for all cases.

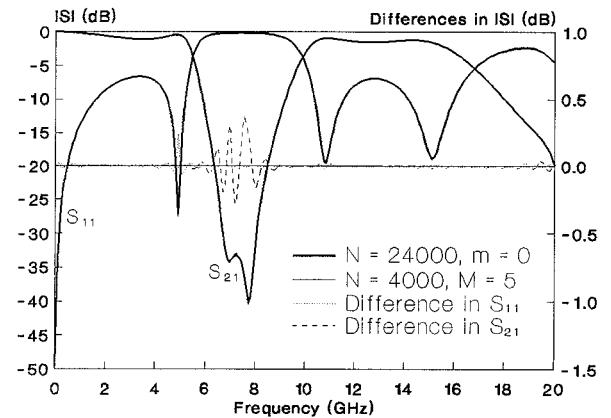


Fig. 2 Comparison between the results obtained with the conventional FDTD/DFT method ( $N = 24000$  and  $m = 0$ ) and the proposed spectral estimation technique ( $N = 4000$  and  $M = 5$ ), where in both cases the frequency increment is the same and it is  $0.094482$  GHz.

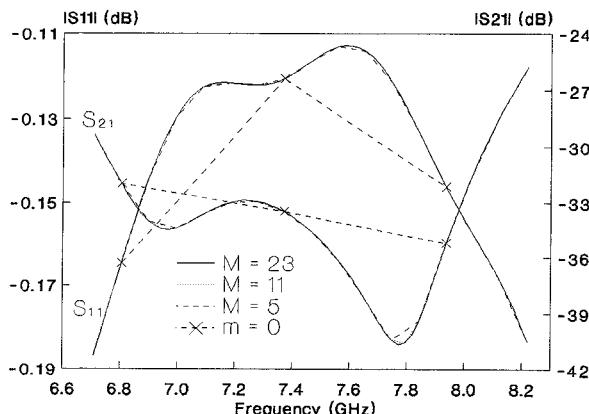


Fig. 3 Convergence of the S-parameters with  $M$  as a parameter in the frequency range of 6.708 GHz - 8.22 GHz.  $N = 4000$  for all cases.